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Research Article

Estimation of Parameter for Disturbance Model of 3-by-3 MIMO Process Using Relay Feedback Test

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Abstract

Ideal and biased relays are used to obtain input/output responses from unknown process systems of 3-by-3 Multiple-Input Multiple-Output (MIMO) nature with component transfer functions of first-order-plus-dead-time (FOPDT) ($D/\tau < 1$) type. To characterize these responses through modeling, a systematic approach is followed to derive analytical expressions for these relay feedback responses. Unknown system parameters are identified using limit cycle data of undesirable relay response curves obtained from single relay feedback test on a linear 3-by-3 MIMO process. Time domain analytical expressions of these monotonic response curves are used to accurately derive boundary conditions and an estimation algorithm for 3-by-3 MIMO process (distillation).

Keywords: Relay feedback; Auto tuning; Multivariable; Modeling; Identification

1. Introduction

Chemical process systems are dominated by time delays, and most of them are multivariable in nature. Interactions exist between inputs and outputs. Controller design (decentralized/centralized) is necessary for increased productivity, safe operation and quality control of the desired outputs. Most of the closed-loops use PID controllers due to their ease in implementation and maintenance. Auto tuning is a recently developed tool to efficiently estimate and tune controller parameters. It has two phases, identification and controller design. A good model identified from process input/output data (near its ultimate frequency) is desired for controller tuning. Proper tuning (model based) of PID loops needs proper identification of process models. IMC-PID tuning needs values of process model parameters. Relay feedback is a promising tool for the identification of process models in real-time. Luyben (2001) discussed a very simple technique that needs one

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additional parameter (other than limit cycle data), namely shape factor, to identify process transfer functions using a single relay feedback test for stable and unstable FOPDT systems. The shapes of relay response curves primarily give an idea of the system category and its order. Thyagarajan and Yu (2003) categorized process models by observing the shapes of relay feedback response (generated from mostly FOPDT processes with different D/τ ratio and higher order systems) and identified the transfer function models. Panda (2006) identified the parameters of SOPDT processes with the help of analytical expressions of relay response along with the boundary conditions. Panda et al. (2011) estimated the parameters of several integrating processes using landmark points information on relay responses. Lee et al. (2010) used two areas of relay feedback response to compute the parametric models. These works are, however, related to single input-single output processes. As most of the process systems are of the MIMO type, analysis of origin of interactions becomes a prime task whereby suitable input-output pairs can be selected for the design of controller. As the off-diagonal closed loop transfer functions contain information on interactions, it is needed to analyze the control system based on time domain characteristics. Shen and Yu (1994) presented relay feedback-based parameter estimation methods for multi-input-multi-output (MIMO) systems. Identification of model parameters can be done using closed-loop data as they are obtained from process industries. Many decentralized controller design methods have been used. These methods consider the desired response (output: for example, y_1 in the case of the 2x2 system) and input (u_1) for system identification and analysis but neither discuss methods to reduce interactions nor give exact analytical expressions for relay responses that can both help analyze the interaction behavior between input/output and provide information regarding closed-loop parameters (PID using model based tuning rules) of the MIMO system. Exact model parameters and information on interactions can be obtained/calculated from mathematical models of relay responses for MIMO systems. Naturally, the system characteristics can be identified using the real-time data of undesirable response, the relay feedback model, and landmark points. Koo et al. (2004) suggested a method of identifying process models while multiloop control systems are being tuned with the relay feedback sequential autotuning method. Selvakumar and Panda (2010) derived analytical expression for relay response (in time domain) of off-diagonal closed-loop (decentralized) transfer function by approximating the denominator dead-time using Pade's method. Most of these above-mentioned methods are limited to SISO techniques, and none of them use information from off-diagonal elements to identify process parameters. Recently, Sujatha and Panda (2012) output pairs derived the relay response model for the 2x2 multivariable system. As the load disturbances drive the system away from its desired behavior, responses are defined as undesired responses. Models of undesirable response are used to analyze interactions and also justify/select input-output pairs for controller design.

In this paper, the relay feedback approach is used to identify model parameters and controller parameters of the 3-by-3 MIMO process. Undesired relay responses are obtained from theoretical equations and validated against the simulated relay response, thereby, parameter estimation algorithms are formulated using landmark points. The objective of the work is as follows: (a) formulation of mathematical model for cyclic relay responses from off-diagonal transfer functions (closed-loop, decentralized) for enhancing interaction analysis and (b) identification of unknown process parameters from undesired relay responses with the help of landmark points.

Relay feedback tests are carried out on 3-by-3 MIMO processes with square matrix structures and having transfer functions of FOPDT nature. Undesirable responses are obtained and modeled for input sensitivity transfer function of 3-by-3 MIMO processes using biased relay feedback tests.

This paper is organized as follows. An introduction on the relay feedback test on MIMO process is detailed in section 2, followed by derivation of analytical expressions of relay responses (from off-diagonal decentralized-closed loop transfer function) for typical 3-by-3 MIMO processes in section 3. In section 4, undesired responses obtained from simulating relay-analytical expressions are validated against that obtained from the experimental (simulink model) response. Parameter estimation from experimental response is presented in section 5 and concluding remarks are given in the final section.

2. Relay Feedback Test Using Sequential Tuning on 3-by-3 MIMO Systems

Based on the sequential auto tuning Shen and Yu (1994) method (as discussed below), each controller is designed in sequence. Let's consider a 3-by-3 MIMO system with a known pairing $(y_1 - u_1)$, $(y_2 - u_2)$ and $(y_3 - u_3)$ under decentralized PI control (Figure 1). Initially, a biased relay is placed between y_1 and u_1, while loops 2 and 3 are on manual. Following the relay-feedback test, a controller can be designed from the ultimate gain and ultimate frequency. The next step is to perform a relay-feedback test between $(y_2 - u_2)$ while loop 1 is on automatic and loop 3 is on manual. Finally, a relay-feedback test between $(y_3 - u_3)$ is performed with loops 1 and 2 on automatic. A controller can also be designed for loop 3 following the relay-feedback test. This procedure is repeated until the controller parameters converge. Typically, the controller parameters converge in 3–4 relay-feedback tests for 3-by-3 MIMO systems.

The 3-by-3 MIMO process with decentralized control structure is shown in Figure 1. 3-by-3 MIMO systems are described by Eq. (2.1) as follows:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \tag{2.1}$$

The decentralized controller is described by Eq. (2.2) as

$$G_c(s) = \begin{pmatrix} g_{c1} & 0 & 0 \\ 0 & g_{c2} & 0 \\ 0 & 0 & g_{c3} \end{pmatrix} \tag{2.2}$$

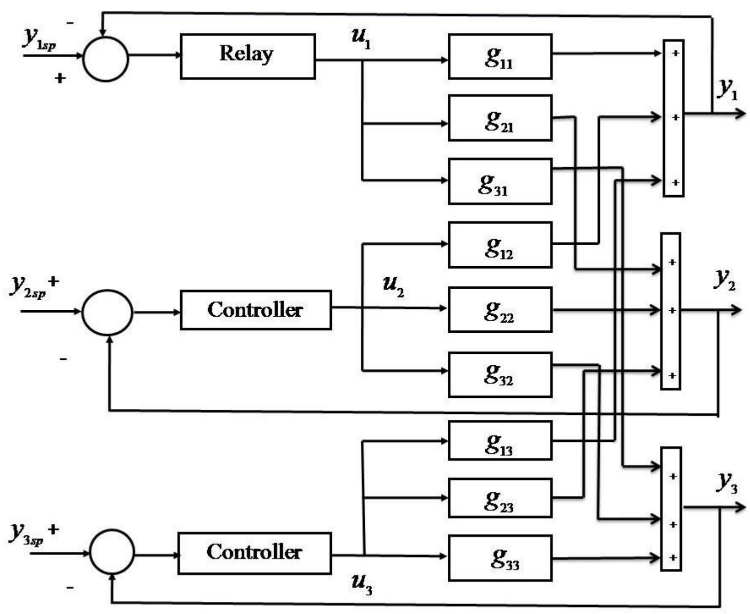

Fig 1. Closed loop representation of 3-by-3 MIMO process with decentralized control structure

As it is evident from closed-loop sequential auto tuning, for a 3-by-3 system, we come across $G_{11,CL}$, $G_{22,CL}$ and $G_{33,CL}$ as diagonal closed-loop transfer functions (that govern basic transfer functions for controller design) and interactive transfer functions as $G_{12,CL}$, $G_{13,CL}$, $G_{21,CL}$, $G_{23,CL}$, $G_{31,CL}$ and $G_{32,CL}$.

The effective closed loop relation between $(y_2 - u_1)$ for 3-by-3 MIMO system is

$$G_{21,CL} = \frac{g_{21}}{1+g_{c2}g_{22}} - \frac{g_{23}g_{31}}{g_{c3}g_{33}\left(1+g_{c3}g_{33}\right)} \tag{2.3}$$

$$g_{21} = \frac{k_{21}e^{-D21s}}{\tau_{21}s+1}\,;\ g_{22} = \frac{k_{22}e^{-D22s}}{\tau_{22}s+1}\,;$$

$$g_{23} = \frac{k_{23}e^{-D23s}}{\tau_{23}s+1}\,;\ g_{31} = \frac{k_{31}e^{-D31s}}{\tau_{31}s+1}\,;\ g_{33} = \frac{k_{33}e^{-D33s}}{\tau_{33}s+1}\,;$$

$$g_{c2} = k_{c2}\left(1+\frac{1}{\tau_{i2}s}\right);\ g_{c3} = k_{c3}\left(1+\frac{1}{\tau_{i3}s}\right)$$

3. Derivation of Analytical Expressions for 3-by-3 MIMO Process

For 3-by-3 MIMO systems, the relay responses (Figure 2a) obtained from interactive transfer function (Eq. (2.3)) are modeled as follows. The biased relay response is assumed to be formed by n-number of infinite-small step changes. Let $\mu_+ = \mu_0 + \mu$, $\mu_- = \mu_0 - \mu$. The process input in the relay feedback test consists of a series of step changes with down amplitude, μ_- and up amplitude, μ_+. At the first interval (after synchronizing input with output by time shift), the response can be described as follows:

$$y_1 = \left(\begin{array}{l} k_{21}\mu_0 \left[1 - e^{-t/\tau_{21}} \right] - \frac{k_{22}k_{c2}}{\tau_{i2}} \mu_- \\ \left[t - (\tau_{i2} + \tau_{22}) \left(1 - e^{-t/\tau_{22}} \right) \right] y_1(t-1) \end{array} \right) - \left(\begin{array}{l} \frac{k_{23}k_{31}\tau_{i3}}{k_{c3}k_{33}} \mu_0 \left[1 - \left(a_1 e^{-t/\tau_{i3}} + a_2 e^{-t/\tau_{23}} + a_3 e^{-t/\tau_{33}} \right) \right] - \\ - \frac{k_{33}k_{c3}}{\tau_{i3}} \mu_- \left[t - (\tau_{i3} + \tau_{33}) \left(1 - e^{-t/\tau_{33}} \right) \right] y_1(t-1) \end{array} \right) \tag{3.1}$$

where
$$a_1 = \frac{\tau_{i3}(\tau_{i3} - \tau_{33})}{(\tau_{i3} - \tau_{23})(\tau_{i3} - \tau_{31})}; a_2 = \frac{\tau_{23}(\tau_{23} - \tau_{33})}{(\tau_{23} - \tau_{i3})(\tau_{23} - \tau_{31})};$$
$$a_3 = \frac{\tau_{31}(\tau_{31} - \tau_{33})}{(\tau_{31} - \tau_{i3})(\tau_{31} - \tau_{23})}$$

Let $D = D_{23} + D_{31} - D_{33}$, at the second instant, the relay output can be given by

$$\begin{array}{l} y_2 = k_{21}\mu_0 \left\{ [1-2] - e^{-\frac{t}{\tau_{21}}} \left[e^{-\frac{D_{21}}{\tau_{21}}} - 2 \right] \right\} - \\ - \frac{k_{22}k_{c2}}{\tau_{i2}} \mu_- \left\{ t[1-2] + \left((\tau_{i2} - \tau_{22}) e^{-\frac{t}{\tau_{22}}} \left[e^{-\frac{D_{22}}{\tau_{22}}} - 2 \right] \right) \right\} y_2(t-1) - \\ - \left(\begin{array}{l} \frac{k_{23}k_{31}\tau_{i3}}{k_{c3}k_{33}} \mu_0 \left[\left\{ 1 - \left(\begin{array}{l} a_1 e^{-\frac{t}{\tau_{i3}}} \left[e^{-\frac{D}{\tau_{i3}}} - 2 \right] + a_2 e^{-\frac{t}{\tau_{23}}} \left[e^{-\frac{D}{\tau_{23}}} - 2 \right] + \\ + a_3 e^{-\frac{t}{\tau_{31}}} \left[e^{-\frac{D}{\tau_{31}}} - 2 \right] \end{array} \right) \right\} \right] - \\ - \frac{k_{33}k_{c3}}{\tau_{i3}} \mu_- \left\{ t[1-2] + \left((\tau_{i3} - \tau_{33}) e^{-\frac{t}{\tau_{33}}} \left[e^{-\frac{D}{\tau_{33}}} - 2 \right] \right) \right\} y_2(t-1) \end{array} \right) \end{array} \tag{3.2}$$

As time tends to infinity, the response becomes stabilized in terms of amplitude and period of oscillation. Thus, the overall response can be described as:

$$y_t = y_1 + y_2 + y_3 + \ldots\ldots\ldots$$

The generalized analytical expressions for a 3-by-3 system is given by:

$$
\begin{aligned}
y_n &= \left(k_{21}\mu_+ - 2k_{21}\mu_0 e^{-\frac{t}{\tau_{21}}} \left(\frac{1-e^{-\frac{p_-}{\tau_{21}}}}{1+e^{-\frac{p_u}{2\tau_{21}}}} \right) \right) - \\
&- \left(\frac{k_{22}k_{c2}}{\tau_{i2}}(\mu\_t_1+\mu t) - \frac{k_{22}k_{c2}}{\tau_{i2}}(\tau_{i2}+\tau_{22}) \left[\mu_+ - 2\mu e^{-\frac{t}{\tau_{22}}} \left(\frac{1-e^{-\frac{p_-}{\tau_{22}}}}{1+e^{-\frac{p_u}{2\tau_{22}}}} \right) \right] \right) y_n(t-1) \\
&- \left(\left[\mu_+ - \left(\begin{array}{l} 2\mu_0 a_1 e^{-t/\tau_{i3}} \left(\frac{1-e^{-\frac{p_-}{\tau_{i3}}}}{1+e^{-\frac{p_u}{2\tau_{i3}}}} \right) + 2\mu_0 a_2 e^{-t/\tau_{23}} \left(\frac{1-e^{-\frac{p_-}{\tau_{23}}}}{1+e^{-\frac{p_u}{2\tau_{23}}}} \right) + \\ +2\mu_0 a_3 e^{-t/\tau_{33}} \left(\frac{1-e^{-\frac{p_-}{\tau_{33}}}}{1+e^{-\frac{p_u}{2\tau_{33}}}} \right) \end{array} \right) \right] - \right. \\
&\left. \left[\frac{k_{33}k_{c3}}{\tau_{i3}}(\mu\_t_1+\mu t) - \frac{k_{33}k_{c3}}{\tau_{i3}}(\tau_{i3}+\tau_{33}) \left[\mu_+ - 2\mu e^{-t/\tau_{33}} \left(\frac{1-e^{-\frac{p_-}{\tau_{33}}}}{1+e^{-\frac{p_u}{2\tau_{33}}}} \right) \right] \right] y_n(t-1) \right)
\end{aligned}
\tag{3.3}
$$

The term $y_n(t-1)$ in equation (3.3) is one step ahead prediction of $y_n(t)$.

4. Validation of Analytical Expressions for 3-by-3 MIMO Systems

The transfer function of multiproduct plant distillation column for the separation of binary mixture of ethanol-water (OR column) is given by

$$
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.61e^{-3.5s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1} \\ \frac{-2.36e^{-3s}}{5s+1} & \frac{-2.3e^{-3s}}{5s+1} & \frac{-0.01e^{-1.2s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.4s}}{10.9s+1} & \frac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{4.1}
$$

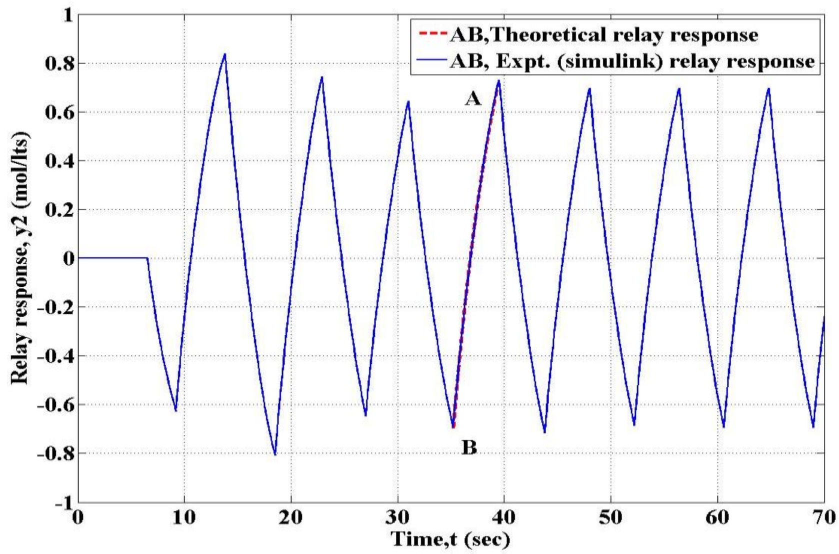

Fig 2a. Validation of analytical expression (dashed line) of 3-by-3 interactive cross TF with simulated relay response (solid line) (T1 column)

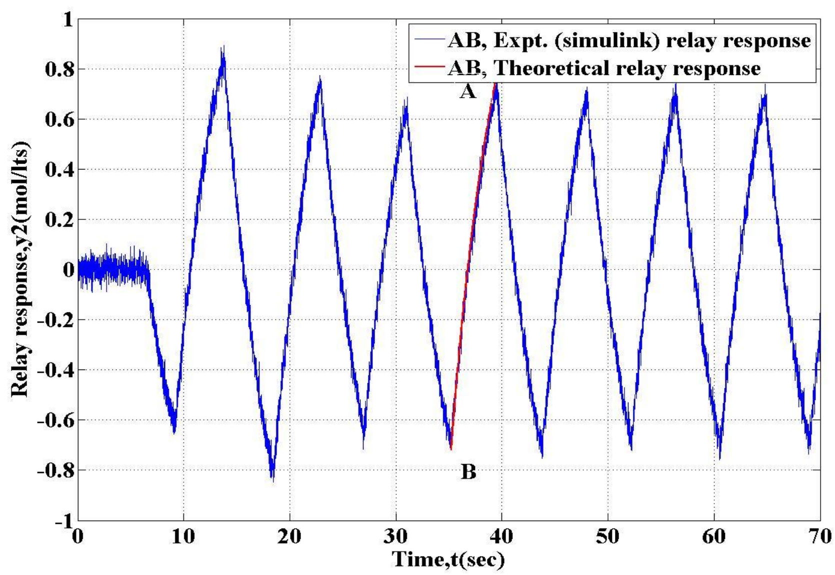

Fig 2b. Validation of analytical expression (dashed line) of 3-by-3 interactive cross TF with simulated relay response (solid line) (T1 column) in the presence of measurement noise.

It can be noted from Eq. (4.1) that $g_{33}(s)$ can be approximated to FOPDT model structure Skogestad (2003) after which the relay response can be generated using simulink (experimental). Similarly, the relay feedback response model given by Eq. (3.3) can be simulated to generate theoretical response (response from undesired-loop is collected) values and is then validated (Figure 2b). Experimental responses were generated using 0.1% measurement noise.

5. Identification with Relay Feedback

Consider the following off-diagonal closed loop transfer function (as given in Eq (2.3)) describing disturbance dynamics:

$$G_{21,CL} = \frac{g_{21}}{1+G_{c2}g_{22}} - \frac{g_{23}g_{31}}{G_{c3}g_{33}\left(1+G_{c3}g_{33}\right)}$$

$$G_{21,CL} = \frac{\dfrac{k_{21}e^{-D21s}}{\tau_{21}s+1}}{1+k_{c2}\left(1+\dfrac{1}{\tau_{i2}s}\right)\dfrac{k_{22}e^{-D22s}}{\tau_{22}s+1}} - \frac{\dfrac{k_{23}e^{-D23s}}{\tau_{23}s+1}\dfrac{k_{31}e^{-D31s}}{\tau_{31}s+1}}{k_{c3}\left(1+\dfrac{1}{\tau_{i3}s}\right)\dfrac{k_{33}e^{-D33s}}{\tau_{33}s+1}\left(1+k_{c3}\left(1+\dfrac{1}{\tau_{i3}s}\right)\dfrac{k_{33}e^{-D33s}}{\tau_{33}s+1}\right)}$$

where (i, j=1,2,3) k_{ii}, k_{ij} are process steady state gain; τ_{ii}, τ_{ij} are process time constant and D, D_{ij} are process dead times. k_{ci}, k_{cj}, τ_{Ii}, τ_{Ij} (i, j=1,2,3) are gain and integral time constant of the controller. The controller transfer functions are supposed to be known while process transfer functions are unknown.

To obtain these 15 unknown process parameters from a single relay experiment, a relay feedback is set up as per the schematic described in Figure 1. The relay response as shown in Figure 2a is used. Hence, disturbance dynamics of 5 processes and 2 controllers of the 3x3 MIMO process are present and inherent in the response. Each process is considered to be of FOPDT model structures to avoid complexity in relay equation. 5 process-dead times are obtained directly from the relay responses. D_{22}, D_{33} are obtained directly from the desired responses y_2, y_3 of loops 2 and 3 respectively. D_{21} is obtained directly from the initial part of the response. Loop 1 is manual and loops 2 and 3 are in auto mode. D_{23} can be obtained from y_2 of loop 2 when loop 3 with y_{3sp} is excited. Similarly, D_{31} can be obtained from y_3 of loop 3 when loop 1 with y_{1sp} is excited.

Out of 15 unknown parameters of disturbance dynamics, 5 dead times are obtained directly from the responses; hence, the remaining ten parameters are estimated based on 10 equations formulated as follows: 6 equations based on landmark points, 4 based on area of relay oscillation, 2 based on moment methods, 1 based on periodic property and 1 based on angle between two lines.

5.1. Landmark Points

Landmark points are the starting point (t=0) and ending point (t=P_u/2), and t_{peak} and D^* can be found easily, as shown in Figure 2b. The boundary conditions are:

$$\left(y_m\right)_{t=0} = -a \tag{5.1}$$

$$\left(y_m\right)_{t=\frac{p_u}{2}} = +a \tag{5.2}$$

$$\left(y_m\right)_{t=\frac{p_u}{2}-D_{21}} = 0 \tag{5.3}$$

$$\left(y_m\right)_{t=\frac{p_u}{2}-D^*} = 0 \tag{5.4}$$

5.2. Area of Relay Oscillation

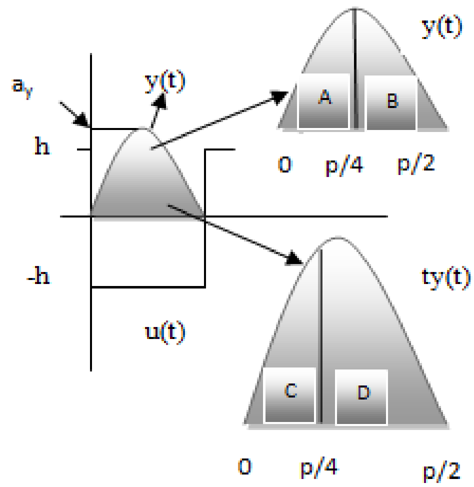

Fig 3. Area between relay feedback response and baseline

Firstly, we consider the following two areas as shown in Figure 3:

$$A = \int_{0}^{p_u/4} y(t)\,dt$$

$$B = \int_{p_u/4}^{p_u/2} y(t)\,dt$$

These areas can describe the integral:

$$q_1 = \int_{0}^{p_u} q\left(t - \frac{p_u}{4}\right) y(t)\,dt = 2\left(\int_{0}^{p_u/4} y(t)\,dt + \int_{p_u/4}^{p_u/2} y(t)\,dt\right)$$

$$q_1 = 2(A + B) \tag{5.5}$$

$$q_2 = \int_{0}^{p_u} q\left(t - \frac{p_u}{4}\right) y(t)\,dt = 2\left(\int_{0}^{p_u/4} y(t)\,dt - \int_{p_u/4}^{p_u/2} y(t)\,dt\right)$$

$$q_2 = 2(A - B) \tag{5.6}$$

In addition to the above relations regarding areas, two more areas considered for better estimates are shown in Figure 3.

$$C = \int_{0}^{p_u/4} ty(t)\,dt$$

$$D = \int_{p_u/4}^{p_u/2} ty(t)\,dt$$

Thus, we can formulate some more relations using the above two expressions for areas:

$$q_3 = \int_{0}^{p_u} r(t)\,y(t)\,dt = 2\left(\int_{0}^{p_u/4} ty(t)\,dt + \int_{p_u/4}^{p_u/2} \left(\frac{p_u}{4} - t\right) y(t)\,dt\right)$$

$$q_3 = 2(C-D) + P_u B \tag{5.7}$$

$$q_4 = \int_0^{p_u} r(t)\,y(t)\,dt = 2\left(\int_0^{p_u/4} \left(\frac{p_u}{4} - t\right) y(t)\,dt + \int_{p_u/4}^{p_u/2} \left(\frac{p_u}{4} - t\right) y(t)\,dt \right)$$

$$q_4 = -2(C+D) + \frac{P_u}{2}(A+B) \tag{5.8}$$

5.3. Moment Method

The moment of a function can be used as a measure to estimate certain parameters of a function. We define the nth moment of a transfer function g (t) about the origin as:

$$m_n = \int_0^{\infty} t^n g(t)\,dt$$

Zeroth moment

$$m_0 = \int_0^{\infty} g(t)\,dt = \lim_{s\to 0} G(s) \tag{5.9}$$

First moment

$$m_1 = \int_0^{\infty} t g(t)\,dt = \lim_{s\to 0} \frac{dG(s)}{ds} \tag{5.10}$$

Using the above-mentioned conditions (5.1 to 5.10) on undesired output response, 10 equations can be synthesized. These equations involving the unknown parameters are formulated using the boundary conditions and solved simultaneously using "fsolve", a command available in the MATLAB toolbox.

Thus, the following steps are followed to estimate parameters of the 3-by-3 MIMO process using lower trenches of relay curves:

(Step 1): Find out D_{21}, D_{23} and D_{31} from initial part of undesirable relay response of 3-by-3 MIMO process.

(Step 2): Find out D_{22}, D_{33} from initial part of desirable relay response of the 3-by-3 MIMO process.

(Step 3): Solve equations (3.5) to (3.14) simultaneously to find unknown parameters:
$k_{21}, \tau_{21}, k_{22}, \tau_{22}, k_{23}, \tau_{23}, k_{31}, \tau_{31}, k_{33}, \tau_{33}$.

After identification, the estimated parameters of the 3-by-3 system are shown as in Table 1.
The efficiency of the present method of estimation in identifying process model (OR process) parameters can be judged by evaluating multiplicative error in Table 2 of the identified model with reference to the real process around its ultimate frequency. This has been reported at a latter section.

Table 1. Comparison of true and estimated process parameters of T1 process

True subsystem	Cheres etal (Open Pass Method) (2003)	Rajapandiyan et al's method (2011)	Proposed method (2012)
$\frac{1.11e^{-6.6s}}{3.25s+1}$	$\frac{1.11e^{-6.49s}}{3.25s+1}$	$\frac{1.1093e^{-6.5012s}}{3.2468s+1}$	$\frac{1.11e^{-6.6s}}{3.249s+1}$
$\frac{-2.36e^{-3s}}{5s+1}$	$\frac{-2.29e^{-3s}}{4.99s+1}$	$\frac{-2.2988e^{-3.009s}}{4.9947s+1}$	$\frac{-2.356e^{-3s}}{5.003s+1}$
$\frac{-0.01e^{-1.2s}}{7.09s+1}$	$\frac{-0.01e^{-1.19s}}{7.09s+1}$	$\frac{-0.01e^{-1.2007s}}{7.0942s+1}$	$\frac{-0.01e^{-1.2s}}{7.09s+1}$
$\frac{-34.68e^{-9.2s}}{8.15s+1}$	$\frac{-34.68e^{-9.18s}}{8.165s+1}$	$\frac{-34.6606e^{-9.1991s}}{8.1438s+1}$	$\frac{-34.68e^{-9.2s}}{8s+1}$
$\frac{0.86e^{-0.88s}}{6.6s+1}$	$\frac{0.86e^{-0.62s}}{6.59s+1}$	$\frac{0.85e^{-0.83s}}{6.5995s+1}$	$\frac{0.859e^{-0.88s}}{6.6s+1}$

Table 2. Multiplicative error for T1 process

Multiplicative error ($P_u = 8.3997$)		
Cheres etal (Open Pass Method) (2003)	Rajapandiyan etal's method (2011)	Proposed method (2012)
0.2842	0.2843	0.2843
1.6661	1.6711	1.7059
7.2002	7.1963	7.2002
12.6951	12.7115	12.9310
5.3794	5.3096	5.3654

6. Conclusions

We have proposed a systematic approach to derive exact expressions for relay feedback responses for interactive transfer functions (closed loop-decentralised) in MIMO systems. The system considered here consists of component transfer functions of FOPDT ($D/\tau < 1$) type with variable time delays. The relay input to linear 3-by-3 MIMO processes produces output with limit cycles which are analyzed to find out ultimate properties of the system. The process output thus obtained can be modeled in time domain as a function of these ultimate system values. A bias/shift value is added to model the response in order to match with the experimental response, and the parameters are estimated based on the shifted model response. A practical example has been considered to rationalize the developed model-equations. When the relay inputs and outputs are synchronized, the response curves help to identify sufficient landmark points that can provide sufficient information for identification of model parameters. Using model-equations, land-mark points, area of relay response, moment method and some curve properties, parameter estimation algorithms have been formulated. Model parameters are identified by solving these estimation algorithms.

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